**Secondary Scheme of Work: Stage 10**

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| *Unit* | *Lessons* | *Key ‘Build a Mathematician’ (BAM) Indicators* | *Essential knowledge* |
| [Investigating properties of shapes](#IPS) | 12 | * [Manipulate fractional indices](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M1_BAM.pdf)
* [Solve problems involving direct and inverse proportion](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M2_BAM.pdf)
* [Convert between recurring decimals and fractions](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M3_BAM.pdf)
* [Solve equations using iterative methods](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M4_BAM.pdf)
* [Manipulate algebraic expressions by factorising a quadratic expression of the form *a*x² + *b*x + *c*](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M5_BAM.pdf)
* [Solve quadratic equations by factorising](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M6_BAM.pdf)
* [Link graphs of quadratic functions to related equations](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M7_BAM.pdf)
* [Interpret a gradient as a rate of change](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M8_BAM.pdf)
* [Recognise and use the equation of a circle with centre at the origin](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M9_BAM.pdf)
* [Apply trigonometry in two dimensions](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M10_BAM.pdf)
* [Calculate volumes of spheres, cones and pyramids](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M11_BAM.pdf)
* [Understand and use vectors](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M12_BAM.pdf)
* [Analyse data through measures of central tendency, including quartiles](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M13_BAM.pdf)
 | * [Know the convention for labelling the sides in a right-angle triangle](http://kangaroomaths.com/free_resources/display/trigonometry.pdf)
* [Know the trigonometric ratios, sinθ = opposite/hypotenuse, cosθ = adjacent/hypotenuse, tanθ = opposite/adjacent](http://kangaroomaths.com/free_resources/display/trigonometry.pdf)
* Know exact values of sinθ and cosθ for θ = 0°, 30°, 45°, 60° and 90°
* Know the exact value of tanθ for θ = 0°, 30°, 45° and 60°
* Know that a^1/n = n√a
* Know that a^-n = 1/an
* Know the information required to describe a transformation
* Know the special case of the difference of two squares
* Know how to set up an equation involving direct or inverse proportion
* Know set notation
* Know the conventions for representing inequalities graphically
* [Know the formulae for the volume of a sphere, a cone and a pyramid](http://www.kangaroomaths.com/free_resources/display/volumes.pdf)
* Know the formulae for the surface area of a sphere, and the curved surface area of a cone
* Know the circle theorems
* Know the characteristic shape of the graph of an exponential function
* Know the meaning of roots, intercepts and turning points
* Know the definition of acceleration
* Know how to construct a box plot
* Know the conditions for perpendicular lines
 |
| [Calculating](#CALC) | 8 |
| [Solving equations and inequalities I](#SEI1) | 9 |
| [Mathematical movement I](#MM1) | 6 |
| [Algebraic proficiency: tinkering](#APT) | 12 |
| [Proportional reasoning](#PR) | 7 |
| [Pattern sniffing](#PS) | 4 |
| [Solving equations and inequalities II](#SEI2) | 6 |
| [Calculating space](#CS) | 10 |
| [Conjecturing](#CON) | 12 |
| [Algebraic proficiency: visualising I](#APV1) | 12 |
| [Exploring fractions, decimals and percentages](#EFDP) | 6 |
| [Solving equations and inequalities III](#SEI3) | 8 |
| [Understanding risk](#UR) | 6 |
| [Analysing statistics](#AS) | 12 |
| [Algebraic proficiency: visualising II](#APV2) | 6 |
| [Mathematical movement II](#MM2) | 4 |
| Total: | 140 | [Stage 10 BAM Progress Tracker Sheet](http://www.kangaroomaths.com/free_resources/planning/stage10_tracker.pdf) |

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| **Maths Calendar** | *Based on 8 maths lessons per fortnight, with at least 35 'quality teaching' weeks per year*  |
| Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | Week 6 | Week 7 | Week 8 | Week 9 | Week 10 | Week 11 | Week 12 | Week 13 |
| [Investigating properties of shapes](#IPS) | [Calculating](#CALC) | [Solving equations and inequalities I](#SEI1) | [Math'l movement I](#MM1) | [Algebraic proficiency: tinkering](#APT) | [Proportional reasoning](#PR) |
| [10M10 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M10_BAM.pdf) | [10M1 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M1_BAM.pdf) |  [10M4 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M4_BAM.pdf) |  | [10M5 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M5_BAM.pdf) | [10M2 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M2_BAM.pdf) |
| Week 14 | Week 15 | Week 16 | Week 17 | Week 18 | Week 19 | Week 20 | Week 21 | Week 22 | Week 23 | Week 24 | Week 25 | Week 26 |
|  | Assessment and enrichment | [Patterns](#PS) | [Solving inequalities](#SEI2) | [Calculating space](#CS) | [Conjecturing](#CON) | [Algebraic proficiency: visualising I](#APV1) |
|  |  |  |  | [10M11 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M11_BAM.pdf) |  | [10M8 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M8_BAM.pdf) |
| Week 27 | Week 28 | Week 29 | Week 30 | Week 31 | Week 32 | Week 33 | Week 34 | Week 35 | Week 36 | Week 37 | Week 38 | Week 39 |
| Assessment | [Exploring FDP](#EFDP) | [Solving equations II](#SEI3) | [Understanding risk](#UR) | [Analysing statistics](#AS) | [Visualising II](#APV2) | [Movement II](#MM2) | Assessment |
|  | [10M3 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M3_BAM.pdf) | [10M6 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M6_BAM.pdf), [10M7 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M7_BAM.pdf) |  | [10M13 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M13_BAM.pdf) | [10M9 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M9_BAM.pdf) | [10M12 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M12_BAM.pdf) |  |

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| *Investigating properties of shapes* | *12 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Properties of Shape progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPropertiesShape.xlsx) |
| * make links to similarity (including trigonometric ratios) and scale factors
* know the exact values of sinθ and cosθ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tanθ for θ = 0°, 30°, 45° and 60°
* know the trigonometric ratios, sinθ = opposite/hypotenuse, cosθ = adjacent/hypotenuse, tanθ = opposite/adjacent
* apply it to find angles and lengths in right-angled triangles in two dimensional figures
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Investigate similar triangles
* Explore trigonometry in right-angled triangles
* Set up and solve trigonometric equations
* Use trigonometry to solve practical problems

**Bring on the Maths: GCSE Higher Shape**Investigating angles: #5, #6, #7, #8, #9 | * Appreciate that the ratio of corresponding sides in similar triangles is constant
* Choose an appropriate trigonometric ratio that can be used in a given situation
* Understand that sine, cosine and tangent are functions of an angle
* Establish the exact values of sinθ and cosθ for θ = 0°, 30°, 45°, 60° and 90°
* Establish the exact value of tanθ for θ = 0°, 30°, 45° and 60°
* Use a calculator to find the sine, cosine and tangent of an angle
* Know the trigonometric ratios, sinθ = opp/hyp, cosθ = adj/hyp, tanθ = opp/adj
* Set up and solve a trigonometric equation to find a missing side in a right-angled triangle
* Set up and solve a trigonometric equation when the unknown is in the denominator of a fraction
* Set up and solve a trigonometric equation to find a missing angle in a right-angled triangle
* Use trigonometry to solve problems involving bearings
* Use trigonometry to solve problems involving an angle of depression or an angle of elevation
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand and work with similar shapes
* Solve linear equations, including those with the unknown in the denominator of a fraction
* Understand and use Pythagoras’ theorem
 | SimilarOppositeAdjacentHypotenuseTrigonometryFunctionRatioSineCosineTangentAngle of elevation, angle of depression**Notation**sinθ stands for the ‘sine of θ’sin-1 is the inverse sine function, and not 1÷ sin | Ensure that all students are aware of the importance of their scientific calculator being in degrees mode.Ensure that students do not round until the end of a multi-step calculationThis unit of trigonometry should focus only on right-angled triangles in two dimensions. The sine rule, cosine rule, and applications in three dimensions are covered in Stage 11.Note that inverse functions are explored in Stage 11.NRICH: [History of Trigonometry](http://nrich.maths.org/6843)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All students explore sets of similar triangles with angles of (at least) 30°, 45° and 60° as an introduction to the three trigonometric ratios**The mnemonic ‘Some Of Harry’s Cats Are Heavier Than Other Animals’ is used to help students remember the trigonometric ratios* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me an angle and its exact sine (cosine / tangent). And another …
* Convince me that you have chosen the correct trigonometric function
* (When exploring sets of similar triangles and working out ratios in corresponding cases) why do you think that the results are all similar, but not the same? Could we do anything differently to get results that are closer? How could we make a final conclusion for each ratio?
 | KM: [From set squares to trigonometry](http://kangaroomaths.com/free_resources/teaching/geometry/set_squares_trigonometry.docx)KM: [Trigonometry flowchart](http://kangaroomaths.com/free_resources/ks4/resources/trig_flowchart.doc)NRICH: [Trigonometric protractor](http://nrich.maths.org/5601)NRICH: [Sine and cosine](http://nrich.maths.org/5671)Hwb: [Greenhouse](https://hwb.gov.wales/Resources/resource/51080f3f-4ee0-4607-9479-94a86226bf9c/en)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M10 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M10_BAM.pdf) | * Some students may not appreciate the fact that adjacent and opposite labels are not fixed, and are only relevant to a particular acute angle. In situations where both angles are given this can cause difficulties.
* Some students may not balance an equation such as sin35 = 4/x correctly, believing that the next step is (sin35)/4 = x
* Some students may think that sin-1θ = 1 ÷ sinθ
* Some students may think that sinθ means sin × θ
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| *Calculating* | *8 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Calculation progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberCalculation.xlsx) |
| * estimate powers and roots of any given positive number
* calculate with roots, and with integer and fractional indices
* calculate exactly with surds
* apply and interpret limits of accuracy, including upper and lower bounds
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Estimate with powers and roots
* Calculate with powers and roots
* Explore the impact of rounding
 | * Estimate squares and cubes of numbers up to 100
* Estimate powers of numbers up to 10
* Estimate square roots of numbers up to 150 and cube roots of numbers up to 20
* Know and use the fact that a-n = 1/an
* Know and use the fact that a1/n = n√a
* Calculate exactly with surds
* Choose the required minimum and maximum values when solving a problem involving upper and lower bounds
* Calculate the upper and lower bounds in a given situation
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Calculate with positive indices using written methods and negative indices in the context of standard form
* Know the multiplication and division laws of indices
* Round to a given number of decimal places or significant figures
* Identify the minimum and maximum values of an amount that has been rounded (to nearest x, x d.p., x s.f.)
 | Power, RootIndex, IndicesStandard formInequalityTruncate, RoundMinimum bound, Maximum boundIntervalDecimal place, Significant figureSurdLimit**Notation**Inequalities: e.g. x > 3, -2 < x ≤ 5 | Surd is derived from the Latin ‘surdus’ (‘deaf’ or ‘mute’). A surd is therefore a number that cannot be expressed (‘spoken’) as a rational number. Calculating with surds includes establishing the rules: $\sqrt{a\pm b} $ $\ne $ $\sqrt{a }\pm \sqrt{b }$, $\sqrt{\frac{a}{b}}= \frac{\sqrt{a}}{\sqrt{b}}$ and $\sqrt{a × b} $ = $\sqrt{a }× \sqrt{b }$If a1/n and *n* is even, then a1/n denotes the principle root - the positive *n*th root.NCETM: [Departmental workshops: Index Numbers](https://www.ncetm.org.uk/resources/13249)NCETM: [Departmental workshops: Surds](https://www.ncetm.org.uk/resources/13234)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pattern sniffing is encouraged to establish the result a0 = 1, a-n = 1/an , i.e.**23 = 2 × 2 × 2 = 8, 22= 2 × 2 = 4, 21= 2, 20= 1, 2-1=* $\frac{1}{2}$ |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a surd. And another. And another …
* When a number ‘x’ is rounded to 1 decimal place the result is 2.5. Jenny writes ’2.45 < x < 2.55’. What is wrong with Jenny’s statement? How would you correct it?
* Always/ Sometimes/ Never: $\sqrt{a+b} $= $\sqrt{a }+ \sqrt{b }$
* Convince me that 2-3 = $\frac{1}{8}$
 | KM: [Maths to Infinity: Standard form](http://kangaroomaths.com/free_resources/infinity/standard_form.xlsm), [Maths to Infinity: Indices](http://kangaroomaths.com/free_resources/infinity/indices.xlsm)KM: [Bounding about](http://www.kangaroomaths.com/free_resources/teaching/number/bounding_about.docx) and [PowerPoint](http://www.kangaroomaths.com/free_resources/teaching/number/bounding_about.pptx)KM: [Calculating bounds: a summary](http://kangaroomaths.com/free_resources/teaching/number/calculating_bounds_summary.docx)NRICH: [Powers and Roots – Short Problems](http://nrich.maths.org/9324)NRICH: [Power Countdown](http://nrich.maths.org/6448)Hwb: [Fibonacci Rectangles 1](https://hwb.gov.wales/Resources/resource/60d9c93e-69f2-4583-a13d-29968169cd74/en), [Fibonacci Rectangles 2](https://hwb.gov.wales/Resources/resource/593f7ad6-798e-4949-a1eb-5c1e1a9b4f2f/en)Hwb: [Motorway roadworks](https://hwb.gov.wales/Resources/resource/f4ea00bc-91e1-4438-a686-da75fda120c2/en)Hwb: [Rhayader has moved](https://hwb.gov.wales/Resources/resource/f6adb157-de34-42da-b868-97c2a918d794/en)Hwb: [Manipulating surds](https://hwb.gov.wales/Resources/resource/b65d1455-f3a6-47ac-8378-fb33e7171d31/en)[Powers of 10](http://www.eamesoffice.com/the-work/powers-of-ten/) (external website)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M1 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M1_BAM.pdf) | * Some students may think that negative indices change the sign of a number, for example 2-1= -2 rather than 2-1= $\frac{1}{2}$
* Some students may think $\sqrt{a\pm b} $= $\sqrt{a }\pm \sqrt{b }$
* Some students may struggle to understand why the maximum bound of a rounded number is actually a value which would not round to that number; i.e. if given the fact that a number ‘x’ is rounded to 1 decimal place the result is 2.5, they might write ’2.45 < x < 2.55’
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| *Solving equations and inequalities I* | *9 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * find approximate solutions to equations numerically using iteration
* solve two linear simultaneous equations in two variables algebraically
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Find approximate solutions to complex equations
* Solve simultaneous equations
* Solve problems involving simultaneous equations
 | * Understand the meaning of an iterative process
* Show that a solution to a complex equation lies between two given values
* Use an iterative formula to find approximate solutions to equations
* Use an iterative formula to find approximate solutions, to a given number of decimal places, to an equation
* Solve two linear simultaneous equations in two variables by substitution
* Solve two linear simultaneous equations in two variables by elimination (multiplication of both equations required)
* Solve two linear simultaneous equations in two variables by elimination (fractional coefficients)
* Derive and solve two simultaneous equations in complex cases
* Interpret the solution to a pair of simultaneous equations
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand the concept of solving simultaneous equations by elimination
* Solve two linear simultaneous equations in two variables in very simple cases (no multiplication required)
* Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required)
 | UnknownSolveSolution setIntervalDecimal searchIterationSimultaneous equationsSubstitutionElimination **Notation**(a, b) for an open interval[a, b] for a closed interval | ‘Interval bisection’ is often an intuitive approach used by pupils when faced with a certain type of problem (see below). ‘Decimal search’ includes ‘trial and improvement’ when the equation is not set to 0.Having been introduced to iterative processes, iteration is explained as a process for finding approximate solutions to non-linear equations. GCSE examples can be found [here](https://drive.google.com/file/d/0B9L2lYGRiK2bczdZaWJOV3ZkR3M/view).Pupils have been introduced to solving simultaneous equations using elimination in simple cases in Stage 9. This includes either no multiplication being required or multiplication of just one equation being required. Solving simultaneous equations using substitution is new to this Stage. NCETM: [Departmental workshops: Simultaneous equations](https://www.ncetm.org.uk/resources/10340)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pupils are taught to label the equations (1) and (2), and label the subsequent equations (3), (4), etc.**Pupils are taught to use the ‘ANS’ key on their calculators when finding an approximate solution using iteration* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a pair of simultaneous equations with a solution x = 4, y = -2. And another. And another …
* Convince me x + 2y = 11, 3x + 4y = 18 can be solved using substitution and using elimination. Which method is best in this case?
* Always/ Sometimes/ Never: Solving a pair of simultaneous equations using elimination is more efficient than using substitution
 | KM: Introduce iterative processes (in this example, interval bisection) by challenging students to find your chosen number (between 1 and 1000000) when the only clue is ‘bigger’ or smaller’ after each guess. Compare the final number of guesses with 20 (since 220 is close to 1000000 and students will probably have very quickly developed a process of roughly bisecting intervals).KM: [Babylonian square roots](http://www.kangaroomaths.com/free_resources/teaching/number/babylonian_square_roots.docx) – an introduction to iterative processesKM: Pre-iterationKM: [Iteration](http://www.kangaroomaths.com/free_resources/teaching/algebra/iteration.docx)KM: [Stick on the Maths: ALG2 Simultaneous linear equations](http://www.kangaroomaths.com/free_resources/teaching/sotm/level7/7alg2_ewb.doc)KM: [Convinced?: ALG2 Simultaneous linear equations](http://www.kangaroomaths.com/free_resources/assessment/app/level7/ch_l7alg2.pdf)NRICH: [Matchless](http://nrich.maths.org/5674)AQA: [Bridging Units Resource Pocket 4](https://allaboutmaths.aqa.org.uk/attachments/5309.pdf) (Skills builder 2 and 3)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M4 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M4_BAM.pdf) | * Some pupils may not check the solution to a pair of simultaneous equations satisfy both equations
* Some pupils may not multiply all coefficients, or the constant, when multiplying an equation
* Some pupils may struggle to deal with negative numbers correctly when adding or subtracting the equations
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| *Mathematical movement I* | *6 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Position and direction progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPositionDirection.xlsx) |
| * identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement (including fractional scale factors)
* make links *between* similarity and scale factors
* describe the changes and invariance achieved by combinations of rotations, reflections and translations
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore enlargement of 2D shapes
* Investigate the transformation of 2D shapes
 | * Use the centre and scale factor to carry out an enlargement of a 2D shape with a fractional scale factor
* Find the scale factor of an enlargement with fractional scale factor
* Find the centre of an enlargement with fractional scale factor
* Solve problems involving similarity
* Perform a sequence of transformations on a 2D shape
* Find and describe a single transformation given two congruent 2D shapes
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Use the centre and scale factor to carry out an enlargement of a 2D shape with a positive integer scale factor
* Use the concept of scaling in diagrams
* Carry out reflection, rotations and translations of 2D shapes
 | Perpendicular bisectorScale FactorSimilarCongruentInvarianceTransformationRotationReflectionTranslationEnlargement | Pupils have identified, described and constructed congruent shapes using rotation, reflection and translation in Stage 7. They have also identified, described and constructed similar shapes using enlargement in Stage 8 and experienced enlarging shapes using positive integer scale factors in Stage 9. NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils should experience using dynamic software (e.g. Autograph) to explore enlargements using fractional scale factors* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a pair of similar shapes. And another. And another …
* Always/ Sometimes/ Never: The resulting image of an enlargement is larger than the original object
* Kenny thinks rotating an object 90° about the origin followed by a reflection in the y-axis has the same effect as reflecting an object in the y-axis followed by a rotation 90° about the origin. Do you agree with Kenny? Explain your answer.
 | KM: [Enlargement 2](http://kangaroomaths.com/free_resources/teaching/geometry/enlargement_2.docx)KM: [Stick on the Maths SSM3: Enlargement (fractional scale factor)](http://www.kangaroomaths.com/free_resources/assessment/app/level7/ch_l7ssm3.pdf)KM: [Stick on the Maths SSM1: Congruence and similarity](http://www.kangaroomaths.com/free_resources/assessment/app/level8/ch_l8ssm1.pdf)NRICH: [Growing Rectangles](http://nrich.maths.org/6923)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may think that the resulting image of an enlargement has to be larger than the original object.
* Some pupils may think that the order of transforming an object does not have an effect on the size and position of the final image.
* Some pupils may link scale factors and similarity using an additive, rather than multiplicative, relationship.
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| *Algebraic proficiency: tinkering* | *12 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * simplify and manipulate algebraic expressions involving algebraic fractions
* manipulate algebraic expressions by expanding products of more than two binomials
* simplify and manipulate algebraic expressions (including those involving surds) by expanding products of two binomials and factorising quadratic expressions of the form x² + *b*x + *c*, including the difference of two squares
* manipulate algebraic expressions by factorising quadratic expressions of the form *ax*² + *bx* + *c*
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Manipulate algebraic fractions
* Manipulate algebraic expressions
 | * Add and subtract algebraic fractions
* Multiply and divide algebraic fractions
* Simplify an algebraic fraction
* Expand the product of three binomials
* Expand the product of two binomials involving surds
* Factorise an expression involving the difference of two squares
* Factorise a quadratic expression of the form *ax*² + *bx* + *c* (*a* is prime)
* Factorise a quadratic expression of the form *ax*² + *bx* + *c* (*a* is composite)
* Identify when factorisation of the numerator and/or denominator is needed to simplify an algebraic fraction
* Simplify an algebraic fraction that involves factorisation
* Change the subject of a formula when more than two steps are required
* Change the subject of a formula when the required subject appears twice
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Calculate with negative numbers
* Multiply two linear expressions of the form (x ± a)(x ± b)
* Factorise a quadratic expression of the form x² + bx + c
* Add, subtract, multiply and divide proper fractions
* Change the subject of a formula when two steps are required
 | EquivalentEquationExpressionExpandLinearQuadraticAlgebraic FractionDifference of two squaresBinomialFactorise**Notation** | Pupils have applied the four operations to proper, and improper, fractions in Stage 7 and factorised quadratics of the form x² + bx + c in Stage 9. Pupils should build on the experiences of using the grid method in Stage 9 to expand products of more than two binomials. Eg (x + 2)(x + 3)(x – 4) = (x2  + 5x + 6)(x – 4) = x3 + x2 – 14x – 24

|  |  |  |  |
| --- | --- | --- | --- |
|  | x2 | +5x | +6 |
| x | x3 | +5x2 | +6x |
| – 4 | –4x2 | –20x | –24 |

Teachers also need to help pupils ‘see’ the difference of two squares by using pictorial representations NCETM: [Algebra](https://www.ncetm.org.uk/resources/43649)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Students are taught to use the grid method in reverse to factorise a quadratic**Students manipulate algebra tiles to explore factoring quadratics**The difference of two squares is explained using visual representation* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * The answer is 2x² + 10x + c. Show me a possible question. And another.
* Kenny simplifies $\frac{3x^{2}+x}{x}$ as 3x2 + 1. Do you agree with Kenny? Explain.
* Convince me that 1032 – 972 = 1200 without a calculator.
* Convince me that 4x2 – 9 ≡ (3x – 2)(3x + 2).
* Jenny thinks that (3x – 2)2 = 3x2 + 12x + 4. Do you agree with Jenny? Explain your answer.
* Convince me that $\frac{2x^{2}+5x+2}{2x+1}$ = x + 2
 | KM: [Simplifying algebraic fractions](http://kangaroomaths.com/free_resources/teaching/algebra/simplifying_algebraic_fractions.docx)KM: [Maths to Infinity: Brackets](http://kangaroomaths.com/free_resources/infinity/brackets.xlsm) and [Quadratics](http://kangaroomaths.com/free_resources/infinity/quadratics.xlsm)KM: [Stick on the Maths: Quadratic sequences](http://kangaroomaths.com/free_resources/teaching/sotm/level7/7alg5_ewb.doc)NRICH: [What’s possible?](http://nrich.maths.org/742)NRICH: [Finding Factors](http://nrich.maths.org/7452)[Algebra Tiles](http://mathbits.com/MathBits/AlgebraTiles/AlgebraTiles/AlgebraTiles.html) (external site)**Learning review** GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M5 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M5_BAM.pdf) | * Once pupils know how to factorise a quadratic expression of the form x² + bx + c they might overcomplicate the simpler case of factorising an expression such as 3x2 + 6x (≡ (3x + 0)(x + 2))
* Some pupils may think that (x + a)2 ≡ x2 + a2
* Some pupils may apply the ‘rules of factorising’ quadratics of the form x² + bx + c to quadratics of the form ax² + bx + c; e.g. 2x2 + 7x + 10 ≡ (2x + 5)(x + 2) because 2 × 5 = 10 and 2 + 5 = 7.
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| *Proportional reasoning* | *7 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Ratio and Proportion progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_RatioProportion.xlsx) |
| * interpret equations that describe direct and inverse proportion
* recognise and interpret graphs that illustrate direct and inverse proportion
* understand that X is inversely proportional to Y is equivalent to X is proportional to 1/Y
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore differences between direct and inverse proportion
* Investigate ways of representing proportion in situation
* Solve problems involving proportion
 | * Interpret graphs and equations that describe direct proportion
* Interpret graphs and equations that describe inverse proportion
* Solve problems involving the combining of ratios
* Solve complex problems combining understanding of fractions, percentages and/or ratio
* Solve more complex problems involving density
* Solve more complex problems involving pressure
* Solve more complex problems involving speed
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know the difference between direct and inverse proportion
* Recognise direct or inverse proportion in a situation
* Know the features of a graph that represents a direct or inverse proportion situation
* Know the features of an expression (or formula) that represents a direct or inverse proportion situation
* Understand the connection between the multiplier, the expression and the graph
 | Direct proportionInverse proportionMultiplier**Notation**$∝$ - ‘proportional to’ | Pupils have solved simple problems involving direct and inverse proportion in Stage 9. This unit focuses on developing a formal algebraic approach, including the use of proportionality constants, to solve direct and inverse proportion problems.NCETM: [Departmental workshops: Proportional Reasoning](https://www.ncetm.org.uk/resources/10334)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils are taught to find a proportionality constant when solving problems* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me an example of two quantities that will be in direct proportion. And another. And another …
* Convince me that this information shows a proportional relationship. What type of proportion is it?

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| 40 | 50 |
| 60 | 75 |
| 80 | 100 |

* Always/Sometimes/Never: X is inversely proportional to Y is equivalent to X is proportional to 1/Y
 | KM: [Graphing proportion](http://kangaroomaths.com/free_resources/teaching/number/graphing_proportion.docx)KM: [Investigating proportionality 2](http://kangaroomaths.com/free_resources/teaching/number/investigating_proportionality_2.docx)KM: [Stick on the Maths NNS1: Understanding Proportionality](http://www.kangaroomaths.com/free_resources/teaching/sotm/level7/7nns1_ewb.doc)KM: [Stick on the Maths CALC1: Proportional Change and multiplicative methods](http://www.kangaroomaths.com/free_resources/teaching/sotm/level7/7calc1_ewb.doc)KM: [Convinced: NNS1: Understanding Proportionality](http://www.kangaroomaths.com/free_resources/assessment/app/level7/ch_l7nns1.pdf)KM: [Convinced: CALC1: Proportional Change and multiplicative methods](http://www.kangaroomaths.com/free_resources/assessment/app/level7/ch_l7calc1.pdf)Hwb: [Inverse or direct?](http://hwb.wales.gov.uk/Resources/resource/6cc89f7e-a5c0-49fd-8a8d-909f38579549/en)NRICH: [In Proportion](http://nrich.maths.org/9268)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M2 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M2_BAM.pdf) | * Some pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to solve problems
* Some pupils may interpret *‘x is inversely proportional to y’* as y=x/k rather than y = k/x
* Some pupils may think that the proportionality constant always has to be greater than 1
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| *Pattern sniffing* | *4 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * deduce expressions to calculate the nth term of quadratic sequences
* recognise and use simple geometric progressions (r^n where n is an integer, and r is a rational number > 0 )
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore quadratic sequences
* Investigate geometric progressions
 | * Find the nth term of a sequence of the form ax2 + b
* Find the nth term of a sequence of the form ax2 + bx + c
* Recognise and describe a simple geometric progression (of the form rn)
* Find the next three terms, or a given term, in a geometric progression
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Find the nth term for an increasing linear sequence
* Find the nth term for an decreasing linear sequence
* Identify quadratic sequences
* Establish the first and second differences of a quadratic sequence
* Find the next three terms in a quadratic sequence
 | Termnth termGenerateQuadraticFirst (second) differenceGeometric Progression**Notation**T(n) is often used to indicate the ‘nth term’ | In Stage 9, pupils recognised and used quadratic sequences. The focus in this stage is finding the nth term for a quadratic sequence and introducing pupils to geometric sequences (r>0).NCETM: [Departmental workshops: Sequences](https://www.ncetm.org.uk/resources/10337)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All students should use a spreadsheet to explore aspects of sequences during this unit. For example, this could be using formulae to continue a given sequence, to generate the first few terms of a sequence from an nth term as entered, or to find the missing terms in sequence.**Ask pupils to repeatedly fold a piece of paper in half as many times as possible as an introduction to geometric sequences.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a geometric progression. And another. And another….
* Show me a quadratic sequence with nth term 3x2 + bx + c. And another. And another….
* Convince me the nth  term of 19, 16, 11, 4, … is 20 – x2.
* Kenny thinks 1, 1, 1, 1, 1, … is an arithmetic sequence. Jenny thinks 1, 1, 1, 1, 1, … is a geometric sequence. Who is correct? Explain your answer.
 | KM: [Sequence plotting](http://kangaroomaths.com/free_resources/teaching/algebra/sequence_plotting.docx). A grid for plotting nth term against term.KM: [Maths to Infinity: Sequences](http://kangaroomaths.com/free_resources/infinity/sequences.xlsm)KM: [Stick on the Maths: Quadratic sequences](http://kangaroomaths.com/free_resources/teaching/sotm/level7/7alg5_ewb.doc)Hwb: [Linear and quadratic sequences](http://hwb.wales.gov.uk/Resources/resource/7548fc9f-26de-4f08-bd57-d6c1f051d879/en)NRICH: [Growing Surprises](http://nrich.maths.org/11212)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some students may think that it is possible to find an nth term for any sequence.
* Some students may think that the second difference (of a quadratic sequence) is equivalent to the coefficient of x2.
* Some students may substitute into ax2 incorrectly, working out (ax)2 instead.
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| *Solving equations and inequalities II* | *6 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * solve linear inequalities in two variables
* represent the solution set to an inequality using set notation and on a graph
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Understand and use set notation
* Solve inequalities
* Represent inequalities on a graph
 | * State the (simple) inequality represented by a shaded region on a graph
* Construct and shade a graph to show a linear inequality of the form y > ax + b, y < ax + b, y ≥ ax + b or y ≤ ax + b
* Construct and shade a graph to show a linear inequality in two variables stated implicitly
* Construct and shade a graph to represent a set of linear inequalities in two variables
* Find the set of integer coordinates that are solutions to a set of inequalities in two variables
* Use set notation to represent the solution set to an inequality
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand the meaning of the four inequality symbols
* Find the set of integers that are solutions to an inequality
* Use set notation to list a set of integers
* Use a formal method to solve an inequality in one variable
* Plot graphs of linear functions stated explicitly
* Plot graphs of linear functions stated implicitly
 | (Linear) inequalityVariableManipulateSolveSolution setIntegerSet notationRegion**Notation**The inequality symbols: < (less than), > (greater than), ≤ (less than or equal to), ≥ (more than or equal to)A graph to represent solutions to inequalities in two variables. A dotted line represents a boundary that is not included. A solid line represents a boundary that is included.Set notation; e.g. {-2, -1, 0, 1, 2, 3, 4} | Pupils have explored the meaning of an inequality and solved linear inequalities in one variable in Stage 9. This unit focuses on solving linear equalities in two variables, representing the solution set using set notation and on a graph Therefore, it is important that pupils can plot the graphs of linear functions, including x = a and y = b.NCETM: [Departmental workshops: Inequalities](https://www.ncetm.org.uk/resources/10333)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All students experience the use of dynamic graphing software, such as Autograph, to represent the solution sets of inequalities in two variables.**Students are taught to manipulate algebraically rather than be taught ‘tricks’. For example, in the case of -2x > 8, students should not be taught to flip the inequality when dividing by -2. They should be taught to add 2x to both sides. Many students will later generalise themselves. Note that with examples such as 5 < 1 – 4x < 21, subtracting 1 from all three parts, and then adding 4x, results in 4 + 4x < 0 < 20 + 4x. This could be broken down into two inequalities to discover that x < -1 and x > -5, so -5 < x < -1. The ‘trick’ results in the more unconventional solution -1 > x > -5.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a pair of integers that satisfy x + 2y < 6. And another. And another …
* Convince me that the set of inequalities x > 0, y > 0 and x + y < 2 has no positive integer solutions.
* Convince me that the set of inequalities x ≥ 0, y > 0 and x + 2y < 6 has 6 pairs of positive integer solutions.
* What is wrong with this statement? How can you correct it?

*The unshaded region represents the* *solution set for the inequalities:* *x < 1, y ≥ 0 and x + y > 6* | KM: [Linear programming with Lego](http://kangaroomaths.com/free_resources/teaching/algebra/linear_programming_lego.docx)KM: [Linear programming](http://www.kangaroomaths.com/free_resources/autograph/020Linear%20Functions/80LinProgProblem.doc) (Autograph)KM: [Stick on the Maths 8: Inequalities](http://www.kangaroomaths.com/free_resources/teaching/sotm/level8/8alg5_ewb.doc)KM: [Convinced?: Inequalities in two variables](http://www.kangaroomaths.com/free_resources/assessment/app/level8/ch_l8alg5.pdf)NRICH: [Which is bigger?](http://nrich.maths.org/7344)Hwb: [How do we know?](http://hwb.wales.gov.uk/Resources/resource/37a4c78b-bf56-4ade-87a8-5333d58b4eb5/en)MAP: [Defining regions using inequalities](http://map.mathshell.org/lessons.php?collection=8&unit=9265)CIMT: [Inequalities](http://www.cimt.plymouth.ac.uk/projects/mepres/allgcse/bkc16.pdf)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may think that it is possible to multiply or divide both sides of an inequality by a negative number with no impact on the inequality (e.g. if -2x > 12 then x > -6)
* Some pupils may think that strict inequalities, such as y < 2x + 3, are represented by a solid, rather than dashed, line on a graph
* Some pupils may shade the incorrect region
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| *Calculating space* | *10 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Measurement and mensuration progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryMeasurementMensuration.xlsx) |
| * calculatesurface area and volume of spheres, pyramids, cones and composite solids
* apply the concepts of congruence and similarity, including the relationships between length, areas and volumes in similar figures
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Calculate surface areas of solids
* Calculate volumes of solids
* Solve problems involving enlargement and 3D shapes
 | * Use Pythagoras’ theorem to find lengths in a pyramid or cone
* Find the surface area of spheres, cones and pyramids
* Find the volume of spheres, cones and pyramids
* Identify how to find the volume or surface area of a composite solid
* Solve practical problems involving the surface area of solids
* Solve practical problems involving the volume of solids
* Understand the implications of enlargement on area
* Understand the implications of enlargement on volume
* Move freely between scale factors for length, area and volume
* Solve practical problems involving length, area and volume in similar figures
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Calculate exactly with multiples of π
* Know and use the formula for area and circumference of a circle
* Know how to use formulae to find the area of rectangles, parallelograms, triangles, trapezia, circles, sectors and
* Know how to find the area of compound shapes
* Know how to find the surface area of a right prism and a cylinder
* Calculate the surface area of a right prism and a cylinder
* Carry out an enlargement
* Find the scale factor of a given enlargement
* Use Pythagoras’ theorem to find missing lengths in right-angled triangles
 | (Composite) solidSphere, Pyramid, ConePerpendicular (height), (slant height)Surface areaVolumeCongruent, congruenceSimilarity, similar shapes, similar figuresEnlarge, enlargementScale factor**Notation**πAbbreviations of units in the metric system: km, m, cm, mm, mm2, cm2, m2, km2, mm3, cm3, km3 | Pupils have previously learnt how to find the surface area of right prisms and cylinders in Stage 9. This unit focuses on finding the volume and surface areas of cones, spheres and pyramids.Pupils also explore congruence and similarity - the use of proportion tables can be helpful to find the multiplier when solving similarity problems such as:

|  |  |  |
| --- | --- | --- |
|  | Shape A | Shape B |
| Known lengths | 6 | 9 |
| Missing lengths | 10 | 15 |
|  | **→ × 1.5 →** |

NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pupils explore the surface area of spheres using oranges (*[*https://www.youtube.com/watch?v=cAxHYFRx1Fs*](https://www.youtube.com/watch?v=cAxHYFRx1Fs) *)**Pupils explore volumes of pyramids by making nets of pyramids and prisms with the same polygonal base and using sand or sugar to compare volumes.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Always/ Sometimes/ Never: The value of the volume of a pyramid is less than the value of the surface area of a pyramid.
* Always/ Sometimes/ Never: The value of the volume of a sphere is less than the value of the surface area of a sphere.
* Convince me that the volume of a pyramid = 1/3 × A × h
* Convince me that 1 m3 = 1 000 000 cm3
 | KM: [Stick on the Maths 8: Congruence and Similarity](http://www.kangaroomaths.com/free_resources/teaching/sotm/level8/8ssm1_ewb.doc)KM: [Convinced? Congruence and Similarity](http://www.kangaroomaths.com/free_resources/assessment/app/level8/ch_l8ssm1.pdf)NRICH: [Surface Area and Volume](http://nrich.maths.org/9740) and [Nicely Similar](http://nrich.maths.org/5635)Hwb: [Summerhouse](http://hwb.wales.gov.uk/Resources/resource/dcc01b2d-ba39-449d-b5a1-a1c13de4c881/en) and [Radiators](http://hwb.wales.gov.uk/Resources/resource/305a31cd-8cec-4f4f-a507-f23b84c7b851/en)OCR: [Congruence Check In](http://www.ocr.org.uk/Images/222124-topic-check-in-9.02-congruence.pdf) and [Similarity Check In](http://www.ocr.org.uk/Images/222125-topic-check-in-9.04-similarity.pdf)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M11 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M11_BAM.pdf) | * Some pupils will work out 4/3 × (π × r)3 when finding the volume of a sphere.
* Some pupils may confuse the concepts of surface area and volume
* Some pupils will work out 4 × (π × r)2 when finding the surface area of a sphere.
* Some pupils may think the volume of a pyramid = ½ × A × h
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| *Conjecturing* | *12 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Properties of Shape progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPropertiesShape.xlsx) |
| * apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Investigate geometric patterns using circles
* Explore circle theorems
* Make and prove conjectures

**Bring on the Maths: GCSE Higher Shape**Investigating angles in circles: #1, #2, #3, #4 | * Create a chain of logical steps to create a proof in a geometrical situation
* Know that ‘the angle in a semicircle is a right angle’
* Know that ‘the angle at the centre is double the angle at the circumference’
* Know that ‘angles in the same segment are equal’
* Know that ‘opposite angles in a cyclic quadrilateral sum to 180°’
* Know that ‘two tangents from an external point are equal in length’
* Know that ‘a radius is perpendicular to a tangent at that point’
* Know that ‘a radius that bisects a chord is perpendicular to that chord’
* Know the alternate segment theorem
* Use a combination of known and derived facts to solve a geometrical problem
* Identify when a circle theorem can be used to help solve a geometrical problem
* Justify solutions to geometrical problems
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know the vocabulary of circles
* Know angle facts including angles at a point, on a line and in a triangle
* Know angle facts involving parallel lines and vertically opposite angles
* Know the properties of special quadrilaterals
 | Radius, radiiTangentChordTheoremConjectureDeriveProve, proofCounterexample**Notation**Notation for equal lengths and parallel linesThe ‘implies that’ symbol (⇒) | Students should also explore the following (paraphrased) circle theorems:* Cyclic Quadrilateral: [GSP](http://kangaroomaths.com/free_resources/teaching/geometry/circle_theorem_cyclic_quadrilateral.gsp), [Word](http://kangaroomaths.com/free_resources/teaching/geometry/circle_theorem_cyclic_quadrilateral.docx)
* Radius and Tangent: [GSP](http://kangaroomaths.com/free_resources/teaching/geometry/circle_theorem_radius_tangent.gsp), [Word](http://kangaroomaths.com/free_resources/teaching/geometry/circle_theorem_radius_tangent.docx)
* Radius and chord:
* Angles in the Same Segment: [GSP](http://kangaroomaths.com/free_resources/teaching/geometry/circle_theorem_same_segment.gsp), [Word](http://kangaroomaths.com/free_resources/teaching/geometry/circle_theorem_same_segment.docx)
* The Angle in the Centre: [GSP](http://kangaroomaths.com/free_resources/teaching/geometry/circle_theorem_subtending.gsp), [Word](http://kangaroomaths.com/free_resources/teaching/geometry/circle_theorem_subtending.docx)
* Two Tangents: [GSP](http://kangaroomaths.com/free_resources/teaching/geometry/circle_theorem_two_tangents.gsp), [Word](http://kangaroomaths.com/free_resources/teaching/geometry/circle_theorem_two_tangents.docx)
* Alternate Segment Theorem: [GSP](http://kangaroomaths.com/free_resources/teaching/geometry/circle_theorem_alternate_segment.gsp), [Word](http://kangaroomaths.com/free_resources/teaching/geometry/circle_theorem_alternate_segment.docx)

NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All students are first introduced to the idea of circle theorems by investigating* [*Thales Theorem*](http://www.kangaroomaths.com/free_resources/teaching/app_imp_thales.docx)*. This is then extended to demonstrate that ‘the angle at the centre is twice the angle at the circumference’**All students are given the opportunity to create and explore dynamic diagrams of different circle theorems.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * How can you use a set square to find the centre of a circle?
* Show me a radius of this circle. And another, and another … (What does this tell you about the lengths? About the triangle?)
* Provide the steps for a geometrical proof of a circle theorem and ask students to ‘unjumble’ them and create the proof, explaining their thinking at each step
* Use the ‘Always / Sometimes / Never’ approach to introduce a circle theorem
 | KM: [Right angle challenge](http://www.kangaroomaths.com/free_resources/teaching/geometry/thales_theorem.pptx)KM: [Thales Theorem](http://www.kangaroomaths.com/free_resources/teaching/app_imp_thales.docx)KM: [6 point circles](http://www.kangaroomaths.com/free_resources/teaching/geometry/6_point_circles.pdf), [8 point circles](http://www.kangaroomaths.com/free_resources/teaching/geometry/8_point_circles.pdf), [12 point circles](http://www.kangaroomaths.com/free_resources/teaching/geometry/12_point_circles.pdf)KM: [Dynamic diagrams](http://kangaroomaths.com/kenny4.php?page=Kshape#gsp)NRICH: [Circle theorems](http://nrich.maths.org/6007)Hwb: [Cadair Idris](http://hwb.wales.gov.uk/Resources/resource/8a4a37a5-b513-403d-a86f-da33d8291947/en)Hwb: [Cyclic quadrilaterals](http://hwb.wales.gov.uk/Resources/resource/e9b4ed76-b009-4f9e-8834-75eb89de3979/en)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some students may think that a cyclic quadrilateral is formed using three points on the circumference along with the centre of the circle
* Some students may not appreciate the significance of standard geometrical notation for equal lengths and angles, and think that lengths / angles are equal ‘because they look equal’
* Some students may not realise that they can extend the lines on diagrams to help establish necessary facts
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| *Algebraic proficiency: visualising I* | *12 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * plot and interpret graphs (including exponential graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
* calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts
* interpret the gradient at a point on a curve as the instantaneous rate of change
* identify and interpret roots, intercepts, turning points of quadratic functions graphically
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore exponential graphs
* Create and use graphs of non-standard functions
* Investigate gradients of graphs
* Find and interpret areas under graphs
* Investigate features of quadratic graphs
 | * Recognise, plot and interpret exponential graphs
* Plot graphs of non-standard functions
* Use graphs of non-standard functions to solve simple kinematic problems
* Recognise that the gradient of a curve is not constant
* Know that the gradient of a curve is the gradient of the tangent at that point
* Calculate the gradient at a point on a curve
* Interpret the gradient at a point on a curve as the instantaneous rate of change
* Interpret the gradient of a chord as an average rate of change
* Solve problems involving the gradients of graphs in context
* Calculate an estimate for the area under a graph, including the area under a speed-time graph as distance
* Solve problems involving the area under graphs in context
* Identify and interpret roots, intercepts and turning points of quadratic functions graphically
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Plot graphs of linear, quadratic, cubic and reciprocal functions
* Interpret the gradient of a straight line graph as a rate of change
* Plot and interpret graphs of kinematic problems involving distance and speed
 | Function, equationLinear, non-linearQuadratic, cubic, reciprocal, exponentialParabola, AsymptoteGradient, y-intercept, x-intercept, rootRate of changeSketch, plotKinematicSpeed, distance, timeAcceleration, deceleration**Notation***y* = *mx* + *c* | Pupils have met plotting graphs of non-standard functions and using graphs of non-standard functions to solve simple kinematic problems in Stage 9. This unit explores and deepens pupils’ understanding of these concepts. However, they do not explicitly plot graphs of exponential functions until Stage 11.This unit also introduces the concept of gradient as an instantaneous change. Drawing tangents at different points on quadratic/cubic graphs and calculating an estimate of their gradients is a very powerful activity for pupils to appreciate the gradient can change.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils use dynamic graphing software, e.g. Autograph and the ‘gradient value/function’, to explore graphs* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a sketch of an exponential graph. And another. And another …
* What is the same and what is different: y = x2, y = 2x, y=1/2x and y = (1/2)x?
* Always/Sometimes/Never: The gradient of a function is constant.
* Sketch a speed/time graph of your journey to school. What is the same and what is different with the graph of a classmate?
 | KM: [Autograph: Pre-Calculus Activity](http://www.kangaroomaths.com/free_resources/autograph/040Intro%20Calculus/05PreCalcActivity3.20.doc)KM: [Autograph: The numerical gradient](http://www.kangaroomaths.com/free_resources/autograph/040Intro%20Calculus/10NumGrad3.20.doc)NRICH: [What’s that graph?](http://nrich.maths.org/7502)Hwb: [The 100m race](http://hwb.wales.gov.uk/Resources/resource/d821f1c8-bb98-4239-aa77-c6a5402dd0d9/en)MAP: [Representing functions of everyday situations](http://map.mathshell.org/download.php?fileid=1740) ILIM: [Interpreting Distance Time Graphs](http://www.nationalstemcentre.org.uk/elibrary/maths/resource/2004/interpreting-distance-time-graphs-with-a-computer-a5)GCSE: [Subject Knowledge Check - Tangents to a curve and Areas under a curve](https://drive.google.com/file/d/0B9L2lYGRiK2bZ0plY0FxZzhsT0U/view)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M8 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M8_BAM.pdf) | * Some pupils may think the graphs of all quadratic functions intercept the x-axis in one or two places.
* Some pupils may think that gradient has the same value for all points for all functions
* Some pupils may join the graph of y = ax (a>1) to the x-axis
* Some pupils think that the horizontal section of a distance time graph means an object is travelling at constant speed.
* Some pupils think that a section of a distance time graph with negative gradient means an object is travelling backwards or downhill.
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| *Exploring fractions, decimals and percentages* | *6 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Fractions, decimals and percentages progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberFDP.xlsx) |
| * change recurring decimals into their corresponding fractions and vice versa
* set up, solve and interpret the answers in growth and decay problems, including compound interest
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore the links between recurring decimals and fractions
* Solve problems involving repeated percentage change
* Solve problems involving exponential growth and decay
 | * Convert a fraction to a recurring decimal
* Convert a recurring decimal of the form $0.\dot{x}$, $0.\dot{x}\dot{y}$, $0.\dot{x}y\dot{z}$ to a fraction
* Convert a recurring decimal of the form $0.0\dot{x}$, $0.0\dot{x}\dot{y}$, to a fraction
* Recognise when a situation involves compound interest
* Calculate the result of a repeated percentage change, including compound interest
* Solve problems involving growth and decay
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Identify if a fraction is terminating or recurring
* Move freely between terminating fractions, decimals and percentages
* Use a multiplier to calculate the result of percentage changes
 | FractionMixed numberTop-heavy fractionPercentage change, percentage increase, percentage increaseCompound interest, Simple interestTerminating decimal, Recurring decimal(Exponential) growth, decay**Notation**Dot notation for recurring decimals; e.g. $0.\dot{x}y\dot{z}=0.xyzxyzxyz…$ and $0.x\dot{y}=0.xyyy…$Note that other notations for recurring decimals are used, for example the vinculum, $0.\dot{x}y\dot{z}=0.\overbar{xyz}$ (USA); parentheses, $0.\dot{x}y\dot{z}=0.(xyz)$ (parts of Europe); the letter ‘R’, 0.xR (upper or lower case) | The diagonal fraction bar (solidus) was first used by Thomas Twining (1718) when recorded quantities of tea. The division symbol (÷) is called an obelus, but there is no name for a horizontal fraction bar.It is useful to start with 1/3 (a fraction and recurring decimal pupils are familiar with) to explain the method: x = 0.33333… ⇒ 10x = 3.33333… ⇒ 9x = 3 and therefore x = 3/9 = 1/3 NRICH: [History of fractions](http://nrich.maths.org/2515)NRICH: [Teaching fractions with understanding](http://nrich.maths.org/2550)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils use the horizontal fraction bar to avoid confusion when fractions are coefficients in algebraic situations**All pupils use dot notation for recurring decimals**All pupils know the recurring decimal for 1/9, 1/90, 1/900 …* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a fraction that can be expressed as a recurring decimal. And another. And another …
* Always/Sometime/Never: If the denominator is odd, the fraction can ve expressed as a recurring decimal
* Convince me 1/7 can be expressed as a recurring decimal
* Convince me 0.9999999999 … = 1
* Kenny thinks that the interest gained when £100 is increased 20% per annum for 4 years can be calculated by multiplying £100 by 2.0736. Do you agree with Kenny? Explain your answer.
 | KM: Investigate fractions connected to cyclic numbers; e.g. the decimal equivalents of sevenths, nineteenths, etc.KM: [Stick on the Maths 8: Recurring decimals and fractions](http://www.kangaroomaths.com/free_resources/teaching/sotm/level8/8nns1_ewb.doc)KM: [Stick on the Maths 8: Repeated Proportional Change](http://www.kangaroomaths.com/free_resources/teaching/sotm/level8/8calc1_ewb.doc)KM: [Convinced?: Recurring decimals and fractions](http://www.kangaroomaths.com/free_resources/assessment/app/level8/ch_l8nns1.pdf)KM: [Convinced?: Repeated Proportional Change](http://www.kangaroomaths.com/free_resources/assessment/app/level8/ch_l8calc1.pdf)NRICH: [Repetitiously](http://nrich.maths.org/1853/index) Hwb: [Borrowing money: APR](http://hwb.wales.gov.uk/Resources/resource/b0571745-fa5f-4303-a5c6-6e94369bce71/en), [Too good to be true!](http://hwb.wales.gov.uk/Resources/resource/e12648d5-ea68-4f26-bbf7-a67db5cb723e/en), [Double your money!](http://hwb.wales.gov.uk/Resources/resource/9d0ae110-d140-4f3c-b8a5-6f35b1917156/en) and [Comparing interest](http://hwb.wales.gov.uk/Resources/resource/225ba756-a1c7-4478-bc8c-e68631fc2b2b/en)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M3 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M3_BAM.pdf) | * Some pupils may incorrectly think 0.111111… = 1/11
* Some pupils may think that an the amount created by increasing a quantity by 5% repeated four times is the same as increasing the quantity by 5% and multiplying that amount by 4.
* Some pupils may think the percentage multiplier for a 20% increase (or decrease) is 0.2
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| *Solving equations and inequalities III* | *8 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * solve quadratic equations algebraically by factorising
* solve quadratic equations (including those that require rearrangement) algebraically by factorising
* find approximate solutions to quadratic equations using a graph
* deduce roots of quadratic functions algebraically
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Solve quadratic equations
* Use graphs to solve equations
 | * Solve a quadratic equation of the form *x*² + *bx* + *c* = 0 by factorising
* Solve a quadratic equation by rearranging and factorising
* Make connections between graphs and quadratic equations of the form *ax*² + *bx* + *c* = 0
* Make connections between graphs and quadratic equations of the form *ax*² + *bx* + *c* = *dx* + *e*
* Find approximate solutions to quadratic equations using a graph
* Deduce roots of quadratic functions algebraically
* Solve problems that involve solving a quadratic equation in context
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Manipulate linear equations
* Factorise a quadratic expression of the form *x*² + *bx* + *c*
* Factorise a quadratic expression of the form *ax*² + *bx* + *c*
* Make connections between a linear equation and a graph
 | (Quadratic) equationFactoriseRearrangeVariableUnknownManipulateSolveDeducex-interceptRoot | Pupils factorise quadratic expressions of the form ax2 + bx + c in Stage 9 (a = 1) and Stage 10. If A × B = 0 then either A = 0 or B = 0 is a fundamental underlying concept to solving quadratic equations when b ≠ 0 and c ≠ 0 by factorising.Pupils should experience solving quadratics with b ≠ 0 and c = 0, such as x2 + 6x = 0, and quadratics with b ≠ 0 and c ≠ 0, such as x2 + 6x + 8 = 0. Pupils may wish to ‘divide both sides by ‘x’ when solving quadratics such as x2 + 6x = 0 without appreciating that x could equal zero.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pupils are taught how to solve quadratics of the form ax*² + *bx* + *c* = 0 when: * b = 0 , b ≠ 0 and c = 0, b ≠ 0 and c ≠ 0

*Pupils are encouraged, whenever possible, to divide a quadratic equation by a common factor to make the factorising process easier, such as 2x2 + 6x + 8 = 0* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a quadratic equation that can be solved by factorising. And another, and another …
* Show me a quadratic equation with one solution x = 3. And another, and another …
* Always/Sometimes/Never: A quadratic equation can be solved by factorising.
* Convince me why you can’t ‘*divide both sides by x’* when solving x2 + 8x = 0
* Kenny is solving x2 + 6x + 8 = 2 as follows:

*(x + 4)(x + 2) = 2 so x + 4 = 2 or x + 2 = 1.**Therefore, x = -2 and x = -1.* * Do you agree with Kenny? Explain your answer.
 | NRICH: [How old am I?](http://nrich.maths.org/631)NRICH: [Golden thoughts](http://nrich.maths.org/271)Hwb: [Algebra Fails](https://hwb-live-storage.s3-eu-west-1.amazonaws.com/07/03/6e/1f/bbd642e183ffbaceb028194d/Q49_Algebra_fails_Resource.pdf?AWSAccessKeyId=AKIAJLXRMQJHU5RYD7ZQ&Expires=1448919073&Signature=OmY9fEVkeSzvX3HfqWH1ZeYmnGQ%3d)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M6 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M6_BAM.pdf), [10M7 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M7_BAM.pdf) | * Some pupils may not appreciate that a quadratic equation must equal zero when solving by factorising
* Some pupils may solve x2 + 8x = 0 by dividing both sides by x to get x + 8 = 0, x = -8.
* Some pupils may forget to divide by the coefficient of x when solving quadratics such as 2x2 + 5x + 2 = 0, i.e. (2x + 1)(x + 2) = 0 so 2x + 1 = 0 or x + 2 = 0 and therefore x = -1 (rather than -½ or x = -2)
* Some pupils may not divide a quadratic equation by a common factor to make the factorising process easier, such as 2x2 + 6x + 8 = 0
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| *Understanding risk* | *6 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Probability progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Probability.xlsx) |
| * apply systematic listing strategies including use of the product rule for counting
* calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams.
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Understand and use the product rule for counting
* Use Venn diagrams to represent probability situations
* Use two-way tables to represent probability situations
* Solve probability problems involving combined events
 | * Apply the product rule for counting
* Understand set notation used with Venn diagrams: ∪, ∩, ∅, ξ
* Use a Venn diagram to calculate theoretical probabilities
* Use a two-way table to sort information in a probability problem
* Use a two-way table to calculate theoretical probabilities
* Calculate conditional probabilities using different representations
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know when to add two or more probabilities
* Know when to multiply two or more probabilities
* Convert between fractions, decimals and percentages
* Use a tree diagram to calculate probabilities of dependent and independent combined events
 | Outcome, equally likely outcomesEvent, independent event, dependent eventTree diagramsTheoretical probability, experimental probabilityRandomBias, unbiased, fairEnumerateSetConditional probabilityVenn diagram**Notation**Set notation: ∪, ∩, ∅, ξP(A) for the probability of event AProbabilities are expressed as fractions, decimals or percentages. They should not be expressed as ratios (which represent odds) or as words | In Stage 9, pupils calculate the probability of independent and dependent combined events using tree diagrams and enumerate sets and combinations of sets systematically, using tree diagrams. This unit has a strong emphasis on the use of Venn diagrams and two-way tables to solve probability problems.Note: A Venn diagram has regions for all possible combinations of groups whether there are elements in those regions or not.An Euler diagram only shows a region if things exist in that region.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)NCETM: [Department Workshops: Sets and Venn Diagrams](https://www.ncetm.org.uk/resources/30989)FMSP: [Set Notation Poster](http://furthermaths.org.uk/files/maths-feast/2015-Entree-Poster.pdf)**Common approaches***Pupils are taught to draw the border around the Venn ‘regions’ to highlight the elements that are not included in the regions.*

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| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me an example of a Venn diagram. And another. And another
* Show me an example of a two-way table. And another. And another
* Always / Sometimes / Never: All the regions of a Venn diagram must be populated
 | CIMT: [Venn Diagrams](http://www.cimt.org.uk/projects/mepres/book7/bk7_1.pdf)OCR: [Check In: Combined Events and Probability Diagrams](http://www.ocr.org.uk/Images/222132-topic-check-in-11.02-combined-events-and-probability-diagrams.pdf)AQA: Bridging Unit: [Set notation, number lines and Venn diagrams](http://allaboutmaths.aqa.org.uk/attachments/5310.pdf)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * When constructing a Venn diagrams for a given situation, some pupils may struggle to distinguish between elements that are included in the intersection of both regions or only in one of the regions
* Some pupils may muddle the conditions for adding and multiplying probabilities
* Some pupils may add the denominators when adding fractions
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| *Analysing statistics* | *12 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Statistics progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Statistics.xlsx) |
| * infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
* construct and interpret diagrams for grouped discrete data and continuous data, i.e. cumulative frequency graphs, and know their appropriate use
* interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data, including box plots
* interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency including quartiles and inter-quartile range
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Construct and interpret cumulative frequency graphs
* Construct and interpret box plots
* Analyse distributions of data sets
 | * Use a sample to infer properties of a population
* Understand the limitations of sampling
* Apply the Petersen capture-recapture method
* Find the quartiles for discrete data sets
* Calculate and interpret the interquartile range
* Construct and interpret a box plot for discrete data
* Use box plots to compare distributions
* Understand the meaning of cumulative frequency
* Complete a cumulative frequency table
* Construct a cumulative frequency curve
* Use a cumulative frequency curve to estimate the quartiles for grouped continuous data sets
* Use a cumulative frequency curve to estimate properties of grouped continuous data sets
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know the meaning of discrete and continuous data
* Interpret and construct frequency tables
* Analyse data using measures of central tendency
 | Categorical data, Discrete dataContinuous data, Grouped dataAxis, axesPopulationSampleCumulative frequencyBox plot, box-and-whisker diagramCentral tendencyMean, median, modeSpread, dispersion, consistencyRange, Interquartile rangeSkewness | In Stage 8, pupils explore how to find the modal class of set of grouped data, the class containing the median of a set of data, the midpoint of a class, an estimate of the mean from a grouped frequency table and an estimate of the range from a grouped frequency tableThis unit builds on the knowledge by exploring measures of central tendency using quartiles and inter-quartile range.Cumulative frequency curves are usually S-shaped, known as an ogive.Box plots are also known as ‘box and whisker’ plots.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***The median is calculated by finding the (n+1)/2 th item and the lower quartile by finding the (n+1)/4 th item unless n is large (n>30). In the case when n>30, n/2 and n/4 can be used to find the median and lower quartile.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a box plot with a large/small interquartile range. And another. And another.
* What’s the same and what’s different: inter-quartile range, median, mean, mode
* Convince me how to construct a cumulative frequency curve
* Always/Sometimes/Never: The median is greater than the inter-quartile range
 | KM: [Stick on the Maths HD1: Statistics](http://www.kangaroomaths.com/free_resources/teaching/sotm/level8/8hd1_ewb.doc), [HD2: Comparing Distributions](http://www.kangaroomaths.com/free_resources/teaching/sotm/level8/8hd2_ewb.doc)KM: [Cumulative Frequency and Box Plots](http://www.kangaroomaths.com/free_resources/ks4/resources/autograph_boxplot.doc)NRICH: [The Live of Presidents](http://nrich.maths.org/11007)NRICH: [Olympic Triathlon](http://nrich.maths.org/8061)NRICH: [Box Plot Match](http://nrich.maths.org/11002)OCR: [Sampling](http://www.ocr.org.uk/Images/282254-foundation-topic-check-in-12.01-sampling.docx), [Analysing Data](http://www.ocr.org.uk/Images/289113-higher-topic-check-in-12.03-analysing-data.docx)Youtube: Johnny Ball estimates the number of [London cabs](https://www.youtube.com/watch?v=tyX79mPm2xY) using the capture-recapture approach**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M13 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M13_BAM.pdf) | * Some pupils may plot the cumulative frequencies against the midpoints or lower bounds of grouped data
* Some pupils may try to construct a cumulative frequency curve by plotting the frequencies against the upper bound of grouped data
* Some pupils may try to construct a cumulative frequency curve by joining the points with straight lines rather than a smooth curve
* Some pupils may forget to add the ‘whiskers’ when constructing a ‘box and whisker’ plot.
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| *Algebraic proficiency: visualising II* | *6 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * use the form *y = mx + c* to identify perpendicular lines
* recognise and use the equation of a circle with centre at the origin
* find the equation of a tangent to a circle at a given point
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Investigate features of straight line graphs
* Know and use the equation of a circle with centre at the origin
* Solve problems involving the equation of a circle
 | * Know that perpendicular lines have gradients with a product of -1
* Identify perpendicular lines using algebraic methods
* Identify the equation of a circle from its graph
* Use the equation of a circle to draw its graph
* Find the equation of a tangent to circle at a given point
* Solve algebraic problems involving tangents to a circle
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Use the form y = mx + cto identify parallel lines
* Rearrange an equation into the form y = mx + c
* Find the equation of a line through one point with a given gradient
* Find the equation of a line through two given points
* Know and apply Pythagoras’ Theorem
 | Function, equationLinear, non-linearParallelPerpendicularGradienty-intercept, x-intercept, rootSketch, plotCentre (of a circle)RadiusTangent**Notation***y* = *mx* + *c* | This unit builds on the graphs of linear functions from Stage 9 including parallel lines.Exploring the equation of circle is new for the pupils and it is important to check students know the definition of a circle (i.e. the locus of points from a fixed point) to help understand how to derive the general formula (x – a)2 + (y – b)2 = r2 by applying Pythagoras’ theorem to find the distance of (x,y), a general point on the circumference of the circle, from (a,b), the centre of a circle with radius r.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf) **Common approaches***All student use dynamic graphing software to explore perpendicular graphs – i.e. plot two perpendicular lines and analyse the relationship between the gradients of the two lines.**Pupils plot points with a ‘x’ and not ‘‘**Pupils draw graphs in pencil* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me the equation of two lines that are perpendicular to each other. And another. And another.
* Convince me the lines y + 0.5x = 7, 6 – x = 2y and 8 + 2y + 4x = 0 are perpendicular to y = 3 + 2x.
* Show me the equation of a circle - what is the centre and radius of the circle? And another. And another.
* True or False? A straight line can intersect a circle at 0, 1 or 2 points.
* Convince me how to find the equation of a tangent to a circle at a given point
 | KM: [The gradient of perpendicular lines](The%20gradients%20of%20perpendicular%20lines)KM: [Introducing the equation of a circle](http://www.kangaroomaths.com/free_resources/autograph/050AdvPure/010EqnCircleLP3.20.doc)KM: [The general equation of a circle](http://www.kangaroomaths.com/free_resources/autograph/050AdvPure/012EqnCircleWS3.20.doc)KM: [The general equation of a circle](http://www.kangaroomaths.com/free_resources/autograph/050AdvPure/012EqnCircleWS3.20.doc)NRICH: [Perpendicular lines](http://nrich.maths.org/5610)NRICH: [At Right Angles](http://nrich.maths.org/6461)FMSP: [Geogebra – Equation of a tangent to a circle](http://tube.geogebra.org/student/b1172723#material/1172583)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M9 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M9_BAM.pdf) | * Some pupils do not rearrange the equation of a straight line correctly to find the gradient of a straight line. For example, they think that the line y – 2x = 6 has a gradient of -2.
* Some pupils may think that gradient = (change in x) / (change in y) when trying to equation of a line through two given points.
* Some pupils may think that the equation of a circle is (x−a)2+(y−b)2 = r
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| *Mathematical movement II* | *4 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Position and direction progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPositionDirection.xlsx) |
| * apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore the concept of a vector
* Solve problems involving vectors
 | * Know and use different notations for vectors, including diagrammatic representation
* Add and subtract vectors
* Multiply a vector by a scalar
* Solve simple geometrical problems involving vectors
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand column vector notation
 | VectorScalarConstantMagnitude**Notation*****a*** (print) and *a* (written) notation for vectors$\vec{AB}$ notation for vectorsColumn vector notation $\left(\begin{matrix}p\\q\end{matrix}\right)$, *p* = movement right and *q* = movement up | In Stage 7, pupils described a translation as a 2D vector. This unit is designed to explore vectors in more detail.Vector is a latin word for ‘carrier, transporter’ derived from veho (‘I carry, I transport, I bear’). Vectors have magnitude and direction.Scalar is from the latin ‘scala’ meaning ‘a flight of steps, stairs, staircase’.Scalars have magnitude but no direction.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches**Pupils either use underline notation, such as *a,*or $\vec{AB}$ notation when writing vectors***.*** |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a pair of values for a and b to satisfy $\left(\begin{matrix}a\\2\end{matrix}\right)$ + 3$\left(\begin{matrix}b\\2\end{matrix}\right)= \left(\begin{matrix}10\\8\end{matrix}\right) .$ And another pair. And another pair.
* If $\vec{OA}$ = **a** and $\vec{OB}$ = **b** , convince me the vector $\vec{AB}$= **b** – **a**
* Always/Sometimes/Never: $\vec{AB}$=$-\vec{BA}$
 | KM: [Vectors](http://www.kangaroomaths.com/free_resources/autograph/010Getting%20Going/205addvectors3.20.doc)NRICH: [Vectors](http://nrich.maths.org/8753)CIMT: [Vectors](http://www.cimt.plymouth.ac.uk/projects/mepres/allgcse/bkc19.pdf)AQA: [Bridging Units: Vectors](http://allaboutmaths.aqa.org.uk/attachments/5627.pdf)**Learning review**GLOWMaths/JustMaths: [Sample Questions Both Tiers](http://justmaths.co.uk/2016/01/03/9-1-exam-questions-by-topic-both-tiers/)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/)KM: [10M12 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/10M12_BAM.pdf) | * Some pupils may try to write column vectors as fractions, i.e. $\left(\frac{1}{2}\right)$ instead of $\left(\begin{matrix}1\\2\end{matrix}\right)$
* If $\vec{OA}$ = **a** and $\vec{OB}$ = **b** , some pupils may calculate the vector $\vec{AB}$as **a** – **b**
* Some pupils may calculate 2$\left(\begin{matrix}a\\b\end{matrix}\right) $as $\left(\begin{matrix}2a\\b\end{matrix}\right)$
 |