*c*

*b*

*a*

Pythagoras’ theorem tells us that $a^{2}+b^{2}=c^{2}$

3, 4 and 5 are known as a **Pythagorean triple** since they are whole numbers that fit the formula:

$$3^{2}+4^{2}=5^{2}$$

There are many ways to find more Pythagorean triples. Here are two of them:

1. The Greek mathematician, **Euclid**, discovered the following method:

Choose $m$ and $n$ such that $m>n>0$

Now work out $a$, $b$ and $c$ as follows:

$$a=m^{2}-n^{2}$$

$$b=2mn$$

$$c=m^{2}+n^{2}$$

Try it out:

* Let $m$ = 4 and $n$ = 2. Work out the values of $a$, $b$ and $c$.
* Check that you have found a Pythagorean triple.
* Now try again using $m$ = 7 and $n$ = 3.

{\displaystyle a=m^{2}-n^{2},\ \,b=2mn,\ \,c=m^{2}+n^{2}}

1. In 1920, the American mathematician, **Leonard Dickson**, published this method:

Choose $r$, $s$ and $t$ such that $r^{2}=2st$

Now work out $a$, $b$ and $c$ as follows:

$$a=r+s$$

$$b=r+t$$

$$c=r+s+t$$

Try it out:

* Let $r$ = 6
* Work out values for $s$ and $t$.
* Work out the values of $a$, $b$ and $c$.
* Check that you have found a Pythagorean triple.
* Now. Let $r$ = 6 again.
* Work out **different** values for $s$ and $t$.
* Work out the values of $a$, $b$ and $c$.
* Check that you have found another Pythagorean triple.
* Repeat for $r$ = 6 again.

Conjecture:

Use either of these methods to find more Pythagorean triples. What do you notice about the triples that you find? Write down any conjectures that you form.

This one is for A Level students …

1. In 1934, Berggren showed the following:

Pick a Pythagorean triple $x,y,z$

Write it as a column vector $\left(\begin{matrix}x\\y\\z\end{matrix}\right)$

Pick any one of the three matrices $\left(\begin{matrix}1&-2&2\\2&-1&2\\2&-2&3\end{matrix}\right)$ , $\left(\begin{matrix}1&2&2\\2&1&2\\2&2&3\end{matrix}\right)$ or $\left(\begin{matrix}-1&2&2\\-2&1&2\\-2&2&3\end{matrix}\right)$

Multiply your chosen matrix by your chosen column vector.

Try it out

* What do you notice about the result?
* Explore further.
* What is a primitive Pythagorean triple?
* What is a tree of primitive Pythagorean triples?